



Rossmoyne Senior High School

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

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In words

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Your name

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Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	92	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (92 Marks)

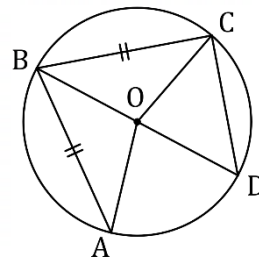
This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(5 marks)

Points A, B, C and D lie on the circle with centre O as shown in the diagram, where $\angle A = 40^\circ$, $AB = BC$ and BD is a diameter.



- (a) Determine the size of $\angle AOD$.

(2 marks)

Solution
Isosceles triangle: $\angle ABO = \angle A = 40^\circ$ Angle on same arc: $\angle AOD = 2 \times 40^\circ = 80^\circ$
Specific behaviours
✓ indicates correct reasoning ✓ calculates angle

- (b) Prove that $\triangle OAD \equiv \triangle ODC$.

(3 marks)

Solution
Angle in semicircle: $\angle BAD = \angle BCD = 90^\circ$ Hence $\triangle BAD \equiv \triangle BCD$ (RHS) and so $CD = AD$ (corresponding sides) Hence $\triangle OAD \equiv \triangle ODC$ (SSS) (No need to show congruency of radii, diameter, etc)
Specific behaviours
✓ establishes a pair of congruent triangles ✓ establishes congruent sides or angles ✓ states appropriate reason for congruency

(b) Prove that $\triangle OAD \equiv \triangle ODC$.

(3 marks)

construct chord AD
 $\angle ABO = 40^\circ$ Base angles of isosceles $\triangle$
 $\angle BOA = 100^\circ$ Angle sum of $\triangle$
 $\angle AOD = 80^\circ$ straight angle
 $\angle BOC = 100^\circ$ Equal chords theorem
 $\angle ODC = 80^\circ$ straight angle
 $OA = OC$ equal radii
 $\angle AOD = \angle ODC = 80^\circ$ ✓
 OD common
 $\therefore \triangle OAD \equiv \triangle ODC$ SAS

Students can start proof here because of part a)

End of questions

Question 10

(5 marks)

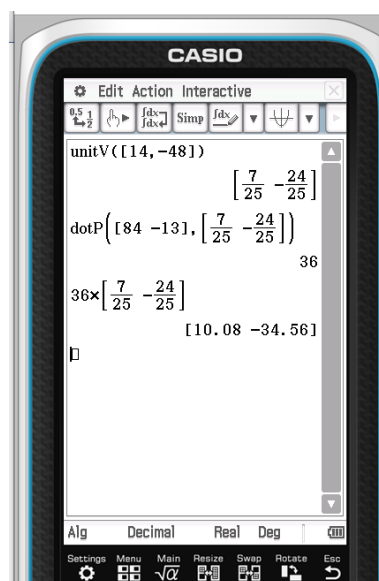
Determine $\mathbf{p}$, the vector projection of

- (a) a force of 320 N on a bearing
- 028°
- onto a force of 300 N on a bearing of
- 345°
- . (3 marks)

Solution
$ \mathbf{p} = 320 \cos 43^\circ = 234 \text{ N}$
Hence $\mathbf{p}$ is a force of 234 N on a bearing of 345° .
Specific behaviours
✓ calculates angle between vectors ✓ calculates magnitude ✓ states direction and magnitude

- (b)
- $\mathbf{m}$
- on
- $\mathbf{n}$
- where
- $\mathbf{m} = (84, -13)$
- and
- $\mathbf{n} = (14, -48)$
- .

(2 marks)



Solution
$\mathbf{p} = \frac{\mathbf{m} \cdot \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}} \mathbf{n}$ $= \frac{1800}{2500} \mathbf{n}$ $= (10.08, -34.56)$
Specific behaviours
✓ indicates method (possibly CAS) Classpad showing SCSA formula use. ✓ calculates vector

Question 11

(8 marks)

- (a) State whether each of the following statements are true or false, supporting each answer with an example or counterexample.

- (i) A polygon with at least six sides will never have an acute interior angle.

(2 marks)

Solution	
False. 	The convex hexagon shown has an acute angle.
Specific behaviours	
✓ states false ✓ sketch with acute angle	

- (ii) If a quadrilateral is not a rectangle then the quadrilateral is not a square. (2 marks)

Solution	
True. 	Squares are rectangles with congruent sides.
Specific behaviours	
✓ states true ✓ sketch of any quadrilateral not a rectangle	

- (b) Consider the statement $\angle P \geq 90^\circ \Rightarrow \angle Q < 90^\circ$ that refers to angles in triangle PQR .

- (i) Write the contrapositive of the statement in simplest form. (1 mark)

Solution
$\angle Q \geq 90^\circ \Rightarrow \angle P < 90^\circ$
Specific behaviours
✓ contrapositive that doesn't use 'not'

- (ii) Write the inverse of the statement in simplest form. (1 mark)

Solution
$\angle P < 90^\circ \Rightarrow \angle Q \geq 90^\circ$
Specific behaviours
✓ correct inverse

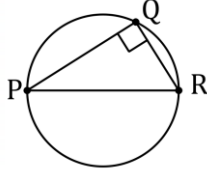
- (iii) Briefly discuss the truth of the original statement, the contrapositive statement, and the inverse statement. (2 marks)

Solution
The original statement is true and so is the contrapositive, by definition. However, the inverse is false - when the triangle is acute, for example.
Specific behaviours
✓ states original and contrapositive true ✓ states inverse false, with justification

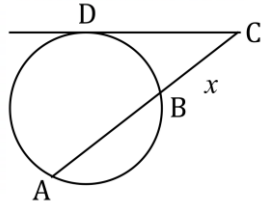
Question 12

(7 marks)

- (a) Points P , Q and R lie on a circle of radius 5 cm, so that PR is a diameter and $QR = 4$ cm. Determine the exact area of triangle PQR . (3 marks)

Solution
 $PR = 10, QR = 4 \Rightarrow PQ = \sqrt{10^2 - 4^2} = \sqrt{84}$ $\text{Area} = \frac{1}{2}(4)2\sqrt{21} = 4\sqrt{21} \text{ cm}^2$
Specific behaviours
✓ indicates PQR is right triangle ✓ calculates missing side ✓ calculates area

- (b) A secant meets a circle at points A and B , where $AB = 4$ cm. A tangent to the same circle at point D intersects the secant at point C , where $CD = 14$ cm. Given that $BC < AC$, determine the exact distance AC and the exact distance BC . (4 marks)

Solution
 Using secant-tangent theorem: $14^2 = x(x + 4)$ $BC = x = 10\sqrt{2} - 2 \text{ cm}$ $AC = BC + 4 = 10\sqrt{2} + 2 \text{ cm}$
Specific behaviours
✓ sketch diagram ✓ formulates equation ✓ solves equation for positive distance ✓ calculates second distance

Question 13

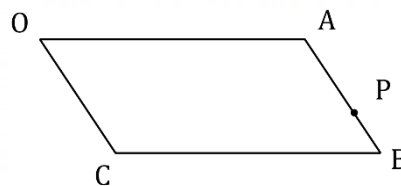
(6 marks)

Parallelogram $OABC$ is shown where point P lies on side AB such that $AP:PB = 2:1$.

Point Q , not shown, lies on diagonal OB such that $OQ:QB = 2:3$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

Express the following in terms of $\mathbf{a}$ and $\mathbf{c}$.



(a) $\overrightarrow{AC}$.

(1 mark)

Solution
$\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$
Specific behaviours
✓ correct expression

(b) $\overrightarrow{OQ}$.

(2 marks)

Solution
$\overrightarrow{OQ} = \frac{2}{5}\overrightarrow{OB} = \frac{2}{5}(\mathbf{a} + \mathbf{c})$
Specific behaviours
✓ uses ratio correctly ✓ correct expression

(c) $\overrightarrow{PQ}$.

(3 marks)

Solution
$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{PB} + \overrightarrow{BQ} \\ &= \frac{1}{3}\mathbf{c} - \frac{3}{5}(\mathbf{a} + \mathbf{c}) \\ &= -\frac{3}{5}\mathbf{a} + \left(\frac{1}{3} - \frac{3}{5}\right)\mathbf{c} \\ &= -\frac{3}{5}\mathbf{a} - \frac{4}{15}\mathbf{c}\end{aligned}$
Specific behaviours
✓ expresses as sum of vectors ✓ expresses individual vectors correctly ✓ simplifies, using correct vector notation throughout

Question 14**(8 marks)**

Consider the set of integers between 1000 and 7000 inclusive.

- (a) Show that there are 546 integers in this set that are a multiple of 11.

(2 marks)

Solution
Number of multiples from 1 to upper bound: $n = \lfloor 7000 \div 11 \rfloor = 636$ Number of multiples from 1 to lower bound: $n = \lfloor 1000 \div 11 \rfloor = 90$ Hence $636 - 90 = 546$ multiples in interval.
Specific behaviours
✓ calculates multiples from 1 to lower, upper bounds ✓ calculates difference

- (b) Determine the number of integers in this set that are

- (i) a multiple of 11 and a multiple of 24.

(3 marks)

Solution
$LCM(11, 24) = 264$ $n = \lfloor 7000 \div 264 \rfloor - \lfloor 1000 \div 264 \rfloor = 26 - 3 = 23$
Specific behaviours
✓ states LCM ✓ calculates multiples from 1 to lower, upper bounds ✓ calculates difference

- (ii) not a multiple of 11 and not a multiple of 24.

(3 marks)

Solution
Multiples of 24: $n = \lfloor 7000 \div 24 \rfloor - \lfloor 1000 \div 24 \rfloor = 291 - 41 = 250$ Multiples of 11 or 24: $n = 546 + 250 - 23 = 773$ Number of integers: $(7000 - 1000 + 1) - 773 = 6001 - 773 = 5228$
Specific behaviours
✓ multiples of 24 ✓ multiples of 11 or 24 ✓ correct number of integers

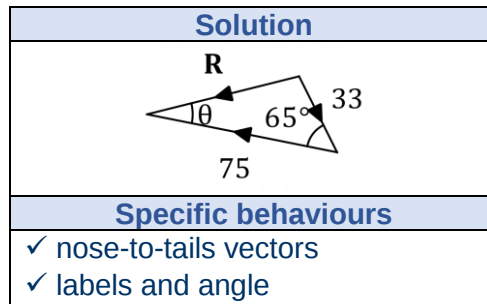
Question 15

(8 marks)

A small body is acted on by force F_1 of 33 N on a bearing of 165° and by force F_2 of 75 N on a bearing of 280° .

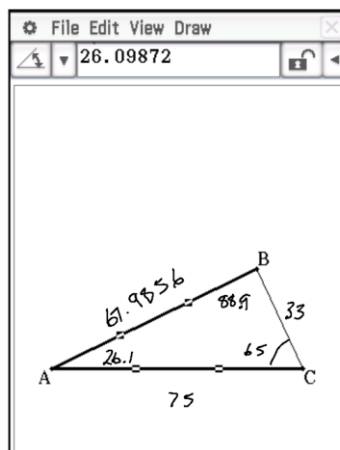
- (a) Sketch a diagram to show $F_1 + F_2$ and their resultant R .

(2 marks)

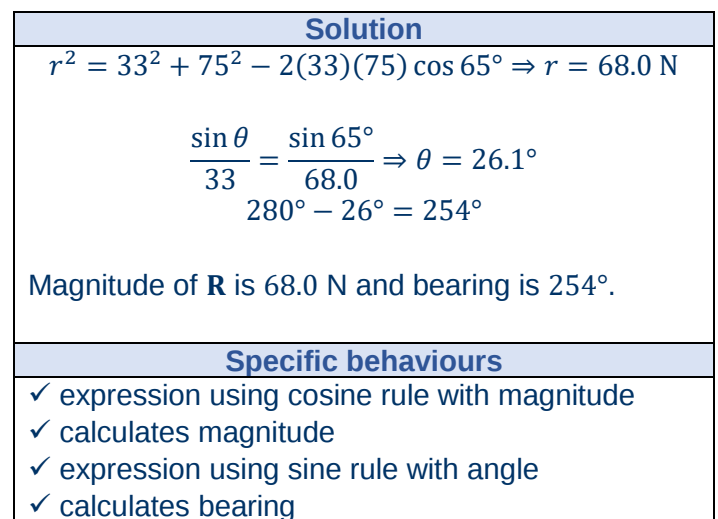


- (b) Determine the magnitude and bearing of R .

(4 marks)

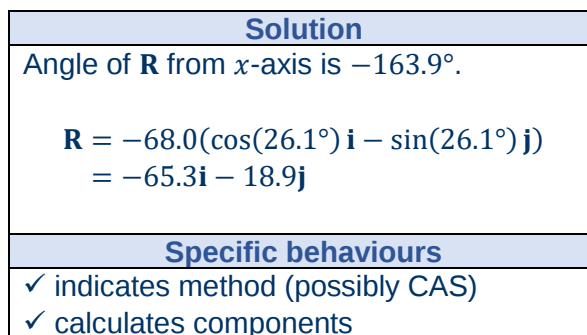


- ✓ sides
- ✓ angles 89, 65
- ✓ state magnitude
- ✓ calculates bearing



- (c) Express R in component form $a\mathbf{i} + b\mathbf{j}$.

(2 marks)

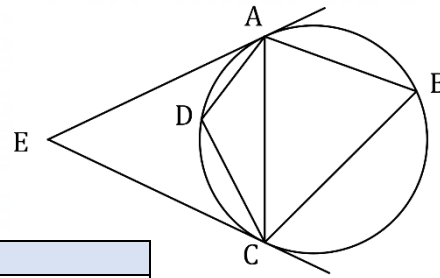


Question 16

(6 marks)

- (a) The diagram shows points A, B, C and D on the circumference of a circle. Tangents to the circle from A and C meet at point E .

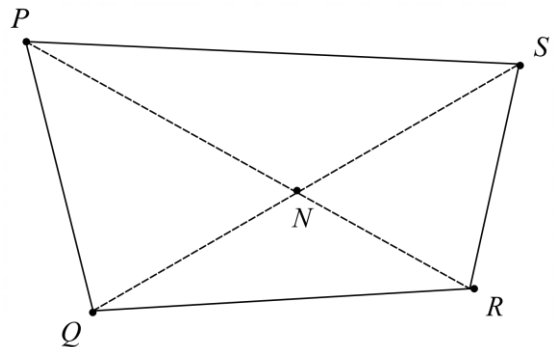
Given that $\angle E = 48^\circ$, determine the size of $\angle B$ and the size of $\angle D$.



(3 marks)

Solution
Isosceles triangle: $\angle EAC = \frac{1}{2}(180^\circ - 48^\circ) = 66^\circ$
Alternate segment: $\angle B = 66^\circ$
Cyclic quadrilateral: $\angle D = 180^\circ - 66^\circ = 114^\circ$
Specific behaviours
✓ shows use of at least two relevant circle theorems ✓ first angle ✓ second angle

- b In quadrilateral $PQRS$, the diagonals PR and SQ intersect at N .



Identify any geometric properties of quadrilateral $PQRS$, if $PN \times NR = SN \times NQ$.

(3 marks)

Solution	Specific behaviours	Point
If $PN \times NR = SN \times NQ$ then $PQRS$ lie on the circumference of a circle Hence opposite angles in the quadrilateral are supplementary.	✓ Recognises intersection of chord theorem applies. ✓ Explains that $PQRS$ lie on the circumference of a circle. ✓ Hence explains why opposite angles are supplementary.	1.3.12

Question 17

(8 marks)

Each letter in the word CLOUDLESS is printed individually on a card. When cards are arranged next to each other in a line, determine the number of different permutations

(a) of all the cards.

(2 marks)

Solution
Note: There are two L's and two S's.
$n = \frac{9!}{2!2!} = 90\,720$
Specific behaviours
✓ expression that allows for repeated letters ✓ calculates number

(b) of all the cards where all the consonants are adjacent.

(2 marks)

Solution
Note: There are six consonants that form a group to be arranged with the remaining three letters.
$n = \frac{4!6!}{2!2!} = 4320$
Specific behaviours
✓ explains or clearly indicates grouping of consonants ✓ calculates number

(c) using any 4 of the cards.

(4 marks)

Solution
Count cases by selecting and then arranging:
1. All letters different
$n_1 = \binom{7}{4} \times 4! = 35 \times 24 = 840$
2. One pair (LL or SS) and two different:
$n_2 = 2 \times \binom{6}{2} \times \frac{4!}{2!} = 30 \times 12 = 360$
3. Two pairs (LL and SS):
$n_3 = 1 \times \frac{4!}{2!2!} = 1 \times 6 = 6$
Number of permutations: $840 + 360 + 6 = 1206$
Specific behaviours
✓ identifies mutually exclusive cases ✓ counts one case correctly ✓ counts second case correctly ✓ counts all cases correctly and calculates total

Question 18

(8 marks)

A circle is drawn with eight points equally spaced on the circumference.

- (a) How many different chords can be drawn between these eight different points? (2 marks)

Solution	Specific behaviours	Point
$\binom{8}{2} = 28$ or $\frac{8 \times 7}{2} = 28$	✓ Correct combination used. ✓ Correct answer. - or ✓ Recognises that 8 points can be selected first, then 7 points. ✓ Divides by 2 as this is counting chords twice, and determines answer.	1.1.7

A student creates triangles by connecting three points on the circumference with straight lines.

- (b) (i) Determine the number of triangles that the student can create. (2 marks)

Solution	Specific behaviours	Point
$\binom{8}{3} = 56$ or $\frac{8 \times 7 \times 6}{3!} = 56$	✓ Correct combination used. ✓ Correct answer - or ✓ 8 choices for first point, then 7, then 6. ✓ Divides by 3! arrangements, and determines answer.	1.1.7

- (ii) How many of the triangles in part (b)(i) will not be right angled? (2 marks)

Solution	Specific behaviours	Point
$4 \times 6 = 24$ right triangles 32 of the triangles are not right angled	✓ Determines there are 4 possible diameters. ✓ Determines number of non-right triangles (consistent with part (b)).	1.1.7 1.3.6

A student creates polygons with three or more sides by connecting the points on the circumference with straight lines.

- (c) Determine the total number of polygons the student can create. (2 marks)

Solution	Specific behaviours	Point
$\binom{8}{3} + \binom{8}{4} + \binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8} = 219$ or $2^8 - \left(\binom{8}{0} + \binom{8}{1} + \binom{8}{2} \right) = 219$	✓ Shows sum of combinations used. ✓ Determines the total number of polygons. - or ✓ Recognises that 2^8 is the total number of ways of choosing 0 to 8 points. ✓ Subtracts combinations that will not produce polygons.	1.1.7

Question 19

(8 marks)

Small bodies P and Q are moving with constant velocities $(2, -2)$ m/s and $(1, 0)$ m/s respectively.

P has initial position vector $(5, 7)$ m and Q has initial position vector $(-3, 13)$ m.

(a) Determine the distance between the bodies after two seconds.

(3 marks)

Solution
Positions after two seconds: $r_P = \begin{pmatrix} 5 \\ 7 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ $r_Q = \begin{pmatrix} -3 \\ 13 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 13 \end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix} -1 \\ 13 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \end{pmatrix} = \begin{pmatrix} -10 \\ 10 \end{pmatrix}$ $ \overrightarrow{PQ} = 10\sqrt{2} \approx 14.14 \text{ m}$
Specific behaviours
✓ positions ✓ vector $\overrightarrow{PQ}$ ✓ distance

(b) Show that the distance between the bodies after t seconds is given by $\sqrt{5t^2 + 40t + 100}$.

(3 marks)

Solution
$r_{PQ} = \begin{pmatrix} -3 \\ 13 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \end{pmatrix} - t \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ $= \begin{pmatrix} -t-8 \\ 2t+6 \end{pmatrix}$ $ r_{PQ} = \sqrt{(-t-8)^2 + (2t+6)^2}$ $= \sqrt{5t^2 + 40t + 100}$
Specific behaviours
✓ vector $\overrightarrow{PQ}$ at time t ✓ simplifies vector ✓ expression for magnitude and simplifies

(c) Prove that the bodies do not meet.

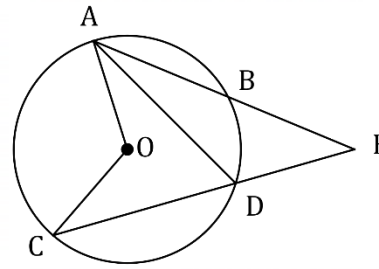
(2 marks)

Solution
Require $5t^2 + 40t + 100 = 0$: $\Delta = b^2 - 4ac = 40^2 - 4(5)(100) = -400$ Since the discriminant is negative, the distance can never be zero and hence the bodies never meet.
Specific behaviours
✓ states condition for bodies to meet ✓ justifies that condition never met

Question 20

(7 marks)

In the diagram shown, secants AB and CD intersect at E , a point outside the circle with centre O .



- (a) Determine the size of $\angle ADC$ and $\angle AOC$ when $\angle E = 28^\circ$ and $\angle EAD = 22^\circ$. (2 marks)

Solution
$\angle ADC = 28^\circ + 22^\circ = 50^\circ$ $\angle AOC = 2 \times 50^\circ = 100^\circ$
Specific behaviours
✓ first angle ✓ second angle

- (b) Prove that when secants AB and CD intersect at E , a point outside the circle with centre O , then $\angle E = \frac{1}{2}(\angle AOC - \angle BOD)$. (4 marks)

Solution
Exterior angle of triangle: $\angle ADC = \angle E + \angle BAD$
Inscribed angles: $\angle ADC = \frac{1}{2}\angle AOC$
Inscribed angles: $\angle BAD = \frac{1}{2}\angle BOD$
Substituting: $\angle E = \frac{1}{2}\angle AOC - \frac{1}{2}\angle BOD$
Factoring: $\angle E = \frac{1}{2}(\angle AOC - \angle BOD)$
Specific behaviours
✓ relation using exterior angles ✓ uses inscribed angles twice ✓ substitutes and factors ✓ notes reasoning throughout

- (c) Determine the size of $\angle E$ when $\angle BOD = 30^\circ$ and $\angle AOC = 80^\circ$. (1 mark)

Solution
$\angle E = \frac{1}{2}(80^\circ - 30^\circ) = 25^\circ$
Specific behaviours
✓ correct angle

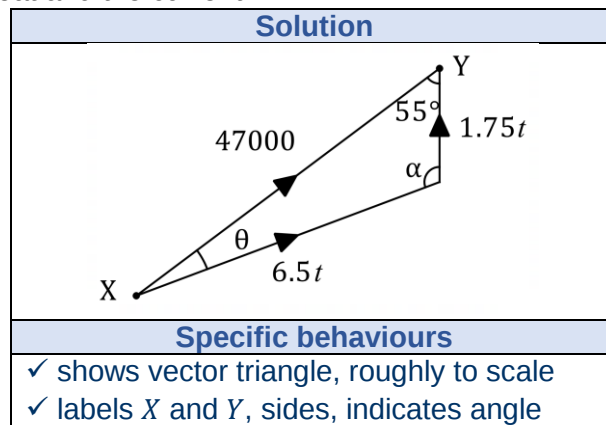
Question 21

(8 marks)

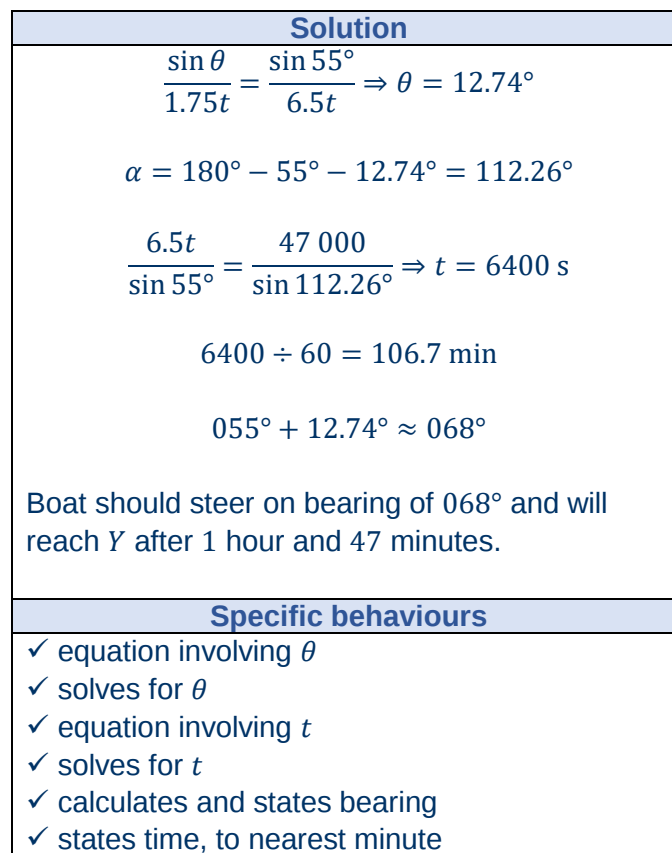
Harbour Y lies on a bearing of 055° from harbour X and the straight line distance between the harbours is 47 km. Between the harbours, a steady current is moving in a northerly direction at a speed of 1.75 metres per second.

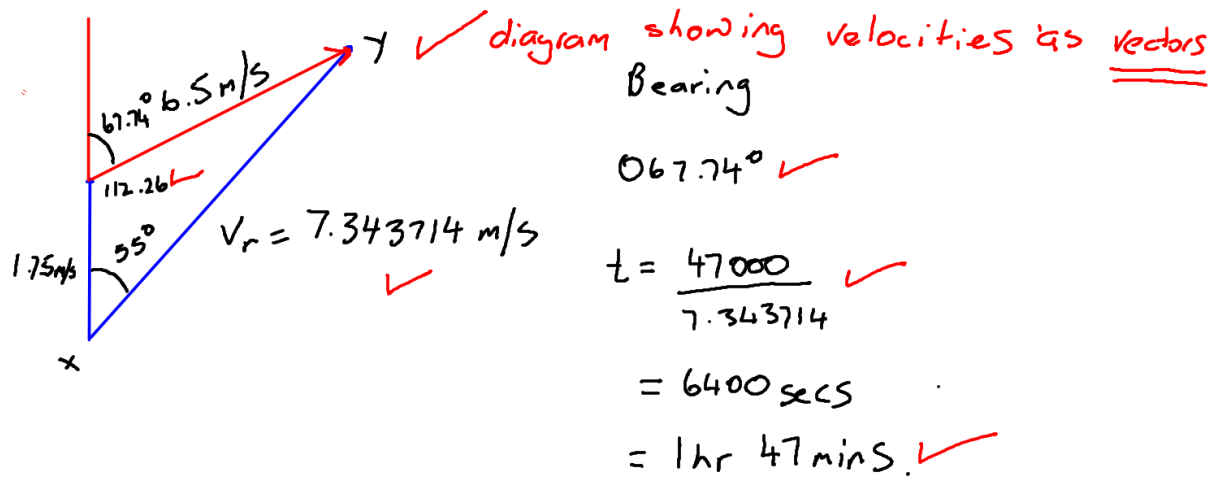
A boat with a cruising speed of 6.5 metres per second is to travel from harbour X to harbour Y in the least possible time.

- (a) Sketch a diagram, roughly to scale, to show the resultant of the sum of the displacement vectors of the boat and the current. (2 marks)



- (b) Determine the bearing it should steer, to the nearest degree, and the time its journey takes, to the nearest minute. (6 marks)





Question number: _____

Supplementary page

Question number: _____

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